Structural Analysis of Helicopter Blades

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Writer's Comment: This report, a response to a Request for Proposal for the Light Helicopter Upgrade Program, was the first project for EAE 135 (Aerospace Structures). Having always been interested in aerospace vehicles, we were glad to be given the opportunity to carry out a structural analysis of the Bell 206 JetRanger's helicopter blades. We learned that structural analysis plays a crucial role in designing safe aerospace vehicles, and we approached this design proposal by learning about the underlying assumptions of a simplified beam theory called "Euler-Bernoulli Beam Theory." From this, we were able to derive the mathematical relationships between displacement of a helicopter blade and the loading under which it was applied, allowing us to compute factors important for selecting the most suitable material. As seniors, we hope to apply our knowledge of material structures to design safe and reliable aircraft for passengers in the future.

Instructor's Comment: Kimberly Jenks and Juanzhu "Christina" Jin are students majoring in Engineering (Kimberly in Mechanical Engineering, Christina in Aerospace Science and Engineering). As their instructor for the course in Aerospace Structures, I am happy to see them receiving such a prestigious campus award. A widespread cliche' for engineering students and engineers is that they do not know how to write and present their ideas. Kimberly and Christina are obviously defying this cliche': they are accomplished writers of a *technically challenging topic, and they were selected among numerous entries across many disciplines.*

– Valeria La Saponara, Department of Mechanical and Aerospace Engineering

1. Introduction

In 2002, a Request for Proposal was announced by the American Helicopter Society to improve the blades of the Bell 206 JetRanger. The Bell 206 is commonly used for news and traffic monitoring. Students were assigned a structural analysis in response to this design proposal for the UC Davis Aerospace Structures course (EAE 135) in Winter 2016. This course uses projects, such as this helicopter-blade analysis, to teach methods for design and rapid prototyping of aerospace components.

Previously, the main helicopter blades were made of Aluminum 7075-T73. The student team was tasked with analyzing the structural performance of blades made with carbon/epoxy. Although Aluminum 7075-T73 is a common aerospace alloy, carbon/epoxy was of interest because of its high stiffness/weight and strength/weight properties, which are particularly important for high performance of helicopters, where weight is critical.

Two carbon/epoxy cases were evaluated: constant cross section and tapered cross section. The Bell 206 JetRanger is shown in Figure 1 along with illustrations of constant and tapered cross sections. As shown, a constant-cross-section blade has the same cross-sectional area throughout its length (x-direction), whereas a tapered-cross-section blade has a cross-sectional area that gradually decreases from blade root to blade tip (also x-direction).

Three helicopter-blade cases were tested for this analysis:

1. Aluminum constant cross section
2. Carbon/epoxy constant cross section
3. Carbon/epoxy tapered cross section

This structural analysis was important because it ensured that failure of helicopter-blade material would not occur during operation. To analyze different materials and geometry, displacement and axial stress due to centrifugal loading were calculated and plotted as functions of span (x-direction). The stress at the blade root (the point at which the blades attach to the main rotor) was subsequently calculated for each blade case. Lastly, the factors of safety at the blade root were determined. In order for an aerospace vehicle to gain flight approval, the vehicle must have a factor of safety of at least 1.5, according to airworthiness requirements of the Federal Aviation Administration (FAA). [1]

The final evaluation accounted for two critical design parameters: safety and weight. Safety was quantified through factor of safety, and weight was calculated from material density. Additional consideration could have been made to strengthen the analysis. For instance, lift could have been included to account for the force keeping the helicopter in the air. This project did not include lift because the main focus was to measure the axial displacement given operating loads. In addition, this project was the first part of a two-part analysis, where lift was considered in the second part. For the purpose of publication, only Part 1 of the two-part analysis was presented here.

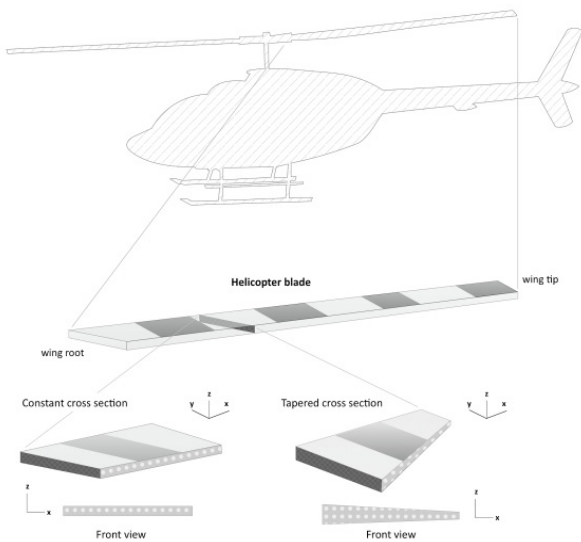

Figure 1: Bell 206 JetRanger with Constant- and Tapered-Cross-Section Blades. [2]

2. Methods

Structural analysis was performed using the Euler-Bernoulli beam theory. [3] This theory converts displacement of material due to loading on a system into a measurement used for determining potential failure during operation. Under this theory, various types of loads can be considered, and several engineering assumptions are made to simplify the calculations. Here, the following assumptions were made:

1. The blade was modeled as a cantilever beam (a beam with one fixed end and one free end). This assumption was made because helicopter blades remain fixed to the main rotor at the blade root and free at the blade tip.
2. The only load considered in this analysis was axial load, which is applied along the longitudinal axis of the helicopter blade (x-axis of Figure 1). The main focus of this project was measurement of the axial displacement caused by the rotational movement of the helicopter blades.
    a. All carbon/epoxy layers were assumed to be 0°-oriented plies under the given loading conditions. The orientation of the carbon/epoxy is an important consideration because material properties depend on direction. For instance, a piece of fabric may be easier to fray in one direction than in another. In the same way, 0°-ply orientation during loading suggests that the blade experiences only axial loading. In reality, the blade could experience additional loads, such as lift (bending) and drag (torsion).
3. The same volume of carbon/epoxy and aluminum are used in the blades. By using the same material volume, the results directly show the effects of changing the blade material and cross-sectional geometry.

Additional assumptions were made by the Euler-Bernoulli beam theory: [3]

1. The blade was assumed to be made of linear-elastic material. This property means that the stress experienced by the helicopter blade increases at the same rate as the blade's displacement (strain). Permanent deformation is not evaluated.

2. The cross section of the blade undergoes rigid-body deformation, which means that the cross section does not change in shape or volume in response to an external load.

The axial load had a value of $F_{axial} = \varrho\Omega 2 x_1 A(x_1)$, where ϱ is the material density, Ω is the angular speed of the blade (rad/s), $A(x_1)$ is the cross-sectional area at location x_1 along the span of the blade. Here, x_1 is the root of the blade. [3] For the remaining equations, x (in reference to Figure 1) will be referred to as x_1. The tapered cross section had a blade-tip cross-sectional area A_1 that was half of the blade-root cross-sectional area A_0 (i.e., $A_1 = (1/2)A_0$). The constant cross section had a cross-sectional area A_0 from blade root to blade tip.

Table 1. Helicopter-Blade Properties

Item	Value
Cross-sectional area of blade root (in^2)	9.648
Cross-sectional area of blade tip (tapered blade) (in^2)	41.469
Blade length (*in*)	194.4
Nominal RPM	396

Table 2. Material Properties of Blades

Material	Item	Value
Carbon/Epoxy [4]	Density (lb/in^3) Young's modulus (lb/in^3) Axial Strength (lb/in^3)	$1.423(10^{-4})$ $21.5(10^6)$ $310(10^3)$
Aluminum 7075-T73 [5]	Density (lb/in^3) Young's modulus (lb/in^3) Axial Strength (lb/in^3)	$2.614(10^{-4})$ $10.4(10^6)$ $60(10^3)$

The flowchart shown in Figure 2 illustrates the path taken to quantitatively conduct the failure analysis. Starting with Block 1, the calculations began with the governing equations describing the system. These equations are used to determine the axial displacement at different points of the system under the applied loads. Shown in Block 2, axial displacement can be subsequently used to calculate the engineering strain (or "deformation"), which is a percentage increase or decrease in length from the system's original length (Equation 1).

$$\epsilon = \frac{\Delta L}{L_{original}} \quad (1)$$

Under the assumption that the material making up the system is linearly elastic, Block 3 shows that the stress can be deduced from the engineering strain. Finally, Block 4 highlights that the failure analysis can be performed by comparing the calculated stress and the material's critical-stress values. This comparison gives the system's overall factor of safety, which is used to verify the system's airworthiness to fly (the factor of safety must be at least 1.5 in order to gain flight approval).

Following Block 1, the governing equation for the displacement of the blade was determined by assuming the blade behaved like a cantilever beam according to the Euler-Bernoulli beam theory. The displacement u1 under the axial load p_1 in the x_1-direction is given in Equation 2. [3]

$$\frac{d}{dx_1}\left(S\frac{du_1}{dx_1}\right) = -p_1(x_1) \quad (2)$$

where S is the axial stiffness of the blade material. Axial stiffness played an important role in determining the most suitable material because larger values of axial stiffness resulted in lower displacement for a given load. Young's modulus *E* was another parameter used to quantify blade performance because higher values of *E* implied greater stiffness and resistance to permanent deformation. For a constant Young's modulus *E* and cross section *A*, the differential equation characterizing the blade displacement u_1 in Equation 3.

$$EA\frac{d^2u_1}{dx_1^2} = -\varrho\Omega^2 x_1 \quad (3)$$

Two boundary conditions were used to solve for the displacement u_1. In general, boundary conditions allow the solution of equations to have actual numerical values, not symbolic values, related to the problem under study. The following boundary conditions were used in this analysis: zero displacement at the blade root, and zero stress at the blade tip. From these boundary conditions, the displacement for the constant-cross-section blade of length *L* is given in Equation 4.

$$u_1(x_1) = \left(\frac{\varrho\Omega^2 L^2}{2E}\right)x_1 - \left(\frac{\varrho\Omega^2}{6E}\right)x_1^3 \quad (4)$$

From a similar analysis, the tapered-cross-section blade was found to have a displacement u_1 given in Equation 5. [3]

$$u_1(x_1) = \frac{\varrho\Omega^2L^3}{2E}\left[\frac{2}{L}x_1 + \frac{1}{2}\left(\frac{x_1}{L}\right)^2 - \frac{1}{3}\left(\frac{x_1}{L}\right)^3 + 2\,ln\left(1 - \frac{x_1}{2L}\right)\right] \quad (5)$$

Now, the governing equations of the problem (Block 1) have been derived. Moving onto Block 2, engineering strain ϵ_1 was determined from the displacement by using kinematic equations. Since axial load is the only load considered in this analysis, engineering strain is referred to as "axial strain," which is the deformation in the x_1-direction (x-direction in Figure 1) shown in Equation 1.

Block 3 follows and stress σ was calculated from axial strain with "constitutive equations." Since the Euler-Bernoulli beam theory was used, the blade material was assumed to be linearly elastic. Therefore, the following relationship described stress σ_1 in terms of axial strain ϵ_1 and Young's modulus E:

$$\sigma_1 = E\epsilon_1 \quad (6)$$

Applying this equation, the stress at the blade root was found to be

$$\sigma_{1,root} = E\epsilon_1|_{x_1=0} \quad (7)$$

where "|" denotes that a parameter is evaluated at a certain value. In this case, ϵ_1 is evaluated at x_1=0. The stresses of the constant- and tapered-cross-section blades were found to be

$$\sigma_{1,root,constant} = \frac{1}{2}\varrho\Omega^2L^2 \quad (8)$$

$$\sigma_{1,root,taper} = \frac{1}{3}\varrho\Omega^2L^2 \quad (9)$$

Finally, Block 4 indicates that the failure analysis was conducted through comparison of the calculated stresses and the material's critical-stress values. In this analysis, the critical-stress values are the yield strengths of the materials because stresses greater than a material's yield strength lead to permanent deformation. For each blade case, the factor of safety (FOS) was computed by relating the calculated stress to the material's yield strength in the following way:

$$FOS = \frac{Yield\ strength\ of\ material}{Calculated\ stress} \tag{10}$$

As previously mentioned, aerospace vehicles must have a factor of safety of at least 1.5 in order to gain flight approval, according to FAA requirements. [1]

3. Results and Discussion

For each blade case, the axial displacement (Block 1) was calculated and plotted against normalized length in Figure 3. Normalized length is used, rather than the actual blade length, to generalize these calculations to all possible blade lengths. The figure shows that the axial displacement of the blade increases from blade root to blade tip.

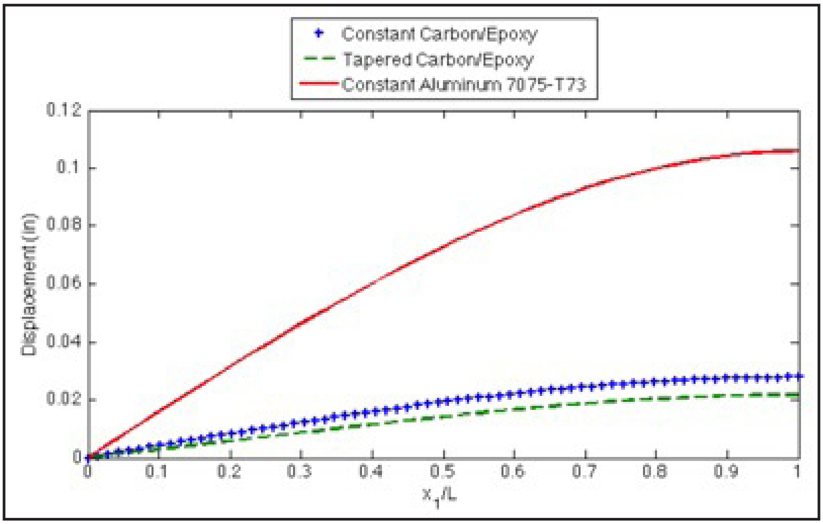

Figure 3: Displacement Versus Normalized Length.

In addition, the aluminum, constant-cross-section blade displays the largest displacement among all cases. The carbon/epoxy, constant-cross-section blade has the second-largest displacement, followed by the carbon/epoxy, tapered-cross-section blade. The carbon/epoxy constant- and tapered-cross-section blades show a small difference. Based on this figure, carbon/epoxy blades displace less than aluminum blades under the same operating condition.

Figure 4 shows that the axial force in the blade decreases from blade root to blade tip. Axial force was of interest because it reflects the stress experienced by the system due to the applied load. If the system undergoes greater stress, the likelihood of failure increases. Figure 4 indicates that an aluminum, constant-cross-section blade experiences the greatest axial force of all blade cases, whereas the carbon/epoxy, tapered-cross-section blade undergoes the least axial force.

Table 3. Root Stresses of Helicopter Blades

Material	Cross-section	Blade-root stress (ksi)
Carbon/Epoxy	Constant	4.625
Carbon/Epoxy	Tapered	3.084
Aluminum	Constant	8.494

Summarized in Table 3, the stress at the blade root was calculated for each blade case. An aluminum, constant-cross-section blade experiences the greatest blade-root stress of 8.494 ksi, while a carbon/epoxy, tapered-cross-section blade undergoes the least blade-root stress of 3.084 ksi. Between the carbon/epoxy blade cases, the constant-cross-section blade shows a higher blade-root stress than the tapered case. This makes sense because a constant-cross-section blade holds more weight than a tapered blade (the cross-sectional area of a tapered blade decreases from blade root to blade tip).

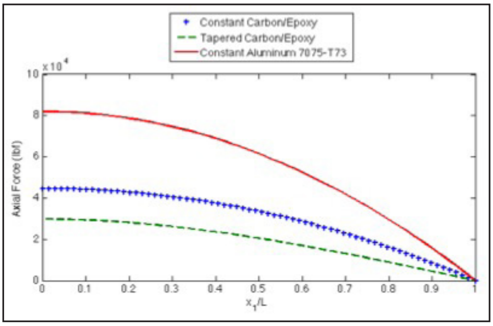

Figure 4: Axial Force Versus Normalized Length

Table 4. Factors of Safety

Material	Cross-section	Factor of Safety
Carbon/Epoxy	Constant	67.02
Carbon/Epoxy	Tapered	100.5
Aluminum	Constant	7.064

Table 4 lists the factor of safety for each blade case. Both carbon/epoxy blades have greater factors of safety than the aluminum blade. This relationship mathematically makes sense because carbon/epoxy is stiffer and stronger than aluminum for the same amount of weight.

Moreover, the carbon/epoxy, tapered-cross-section blade has a higher factor of safety than the carbon/epoxy, constant case. As mentioned, the tapered blade carries less weight than the constant blade, which implies that the tapered blade experiences less stress at the blade root for a given load. Lower stress results in lower likelihood of failure, which increases factor of safety.

Both carbon/epoxy and aluminum blades have factors of safety greater than 1.5, indicating a safe use of the materials and confirmed FAA airworthiness. [1] Since higher factors of safety suggest higher tolerance to cracking and bending, under the operating loads the carbon/epoxy, constant- and tapered-cross-section blades are shown to be safer than the aluminum, constant-cross-section blade.

The integrity of this structural analysis could be improved by addressing the initial assumptions. In particular, the analysis could include multiple loads, including lift, bending, and torsion. The addition of these other loads would not only increase the stress experienced by the blade, but would also alter the carbon/epoxy material properties, which depend on loading direction.

Further, the carbon/epoxy blades could be made more realistic by decreasing the cross-sectional area (Assumption 3). Because of its greater strength/weight and stiffness/weight properties with respect to aluminum, carbon/epoxy is used in weight-sensitive aerospace applications, particularly helicopters.

4. Conclusion

This analysis evaluated the structural integrity of constant- and tapered-cross-section helicopter blades made with carbon/epoxy and aluminum. The Euler-Bernoulli beam theory was used to analyze the displacements, axial forces, engineering strain, and stresses experienced by the blades. The axial forces were shown to decrease from blade root to blade tip, and the carbon/epoxy, constant-cross-section blade experienced greater axial force than the carbon/epoxy, tapered blade. The aluminum, constant-cross-section blade experienced the greatest axial force of all blade cases. The stresses at the blade root were higher for the constant-cross- section blades (4.625-8.494 ksi) than for the tapered blade (3.084 ksi). Finally, the factors of safety for all blade cases were greater than 1.5 (7.064-100.5). As a result, the carbon/epoxy blades were shown to perform more safely under the operating conditions than the aluminum blade. Analysis could be further improved by accounting for multiple loads, including lift (bending) and drag (torsion).

5. Acknowledgements

We would like to thank Professor Valeria La Saponara and Teaching Assistant Alan Carey for their patience and guidance. We appreciate their comments and suggestions throughout the revision process.

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